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14. ABSTRACT The basic two cell qubit contains four Josephson junctions, two residing on the conductor common to the two cell loops, and one on each loop's perimeter. In the limit of zero inductance the qubit potential is described by two phase variables and two orthogonal external magnetic flux parameters (the symmetric and antisymmetric combinations of external flux). The barrier height of the effective double well potential is controlled by the symmetric external flux and the potential tilt by the antisymmetric flux. We will discuss the effect of non-zero inductance and defects on the qubit energy levels, circulating currents, matrix elements, and on the relaxation and phase decoherence times.					
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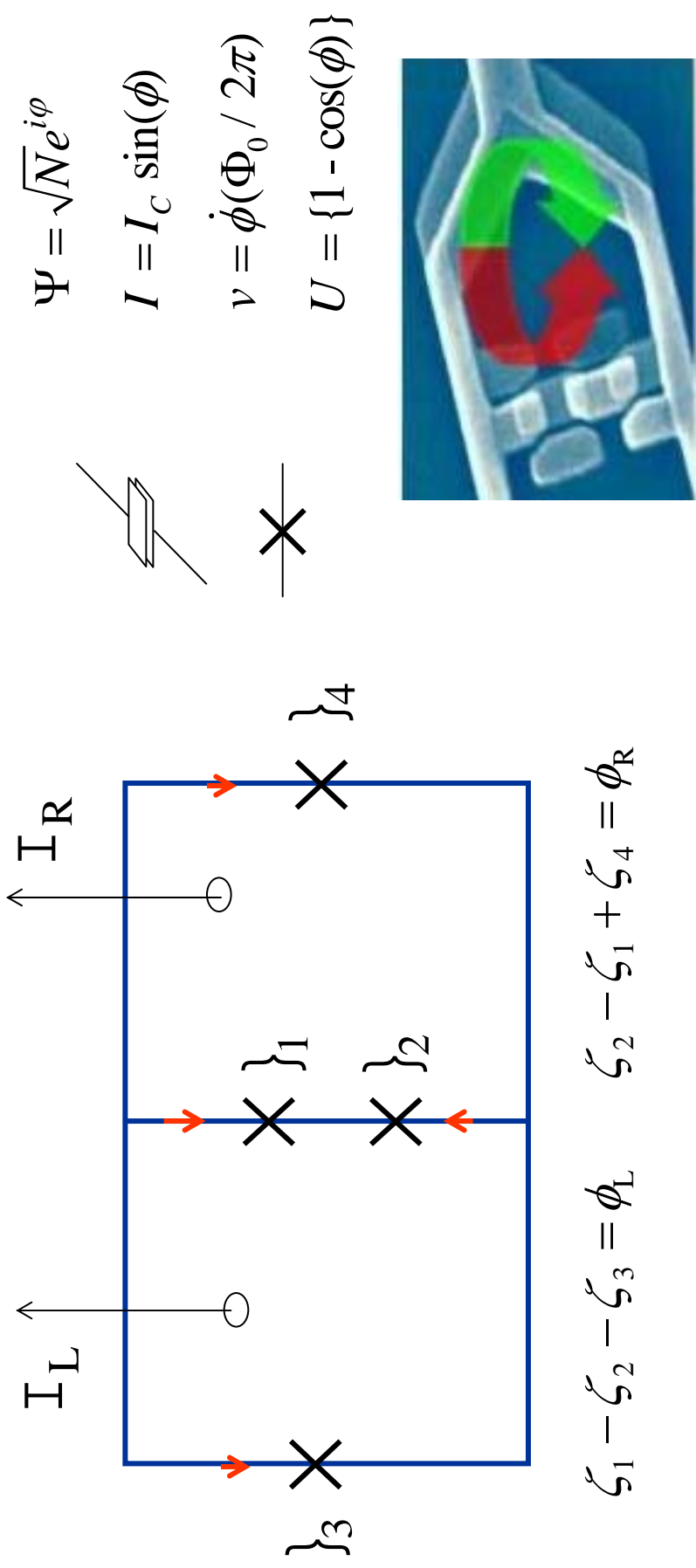
Two Cell Josephson Junction Qubits

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Outline of Talk

- Two Cell Josephson Junction Qubit
- Molmer Sorensen Bichromatic Gates
- Tavis Cummings Hamiltonian
- Forming a Network of LC Buses
- Experimental Developments
- Conclusions

Potential $V(\theta, \psi)$ for a Two Cell Qubit

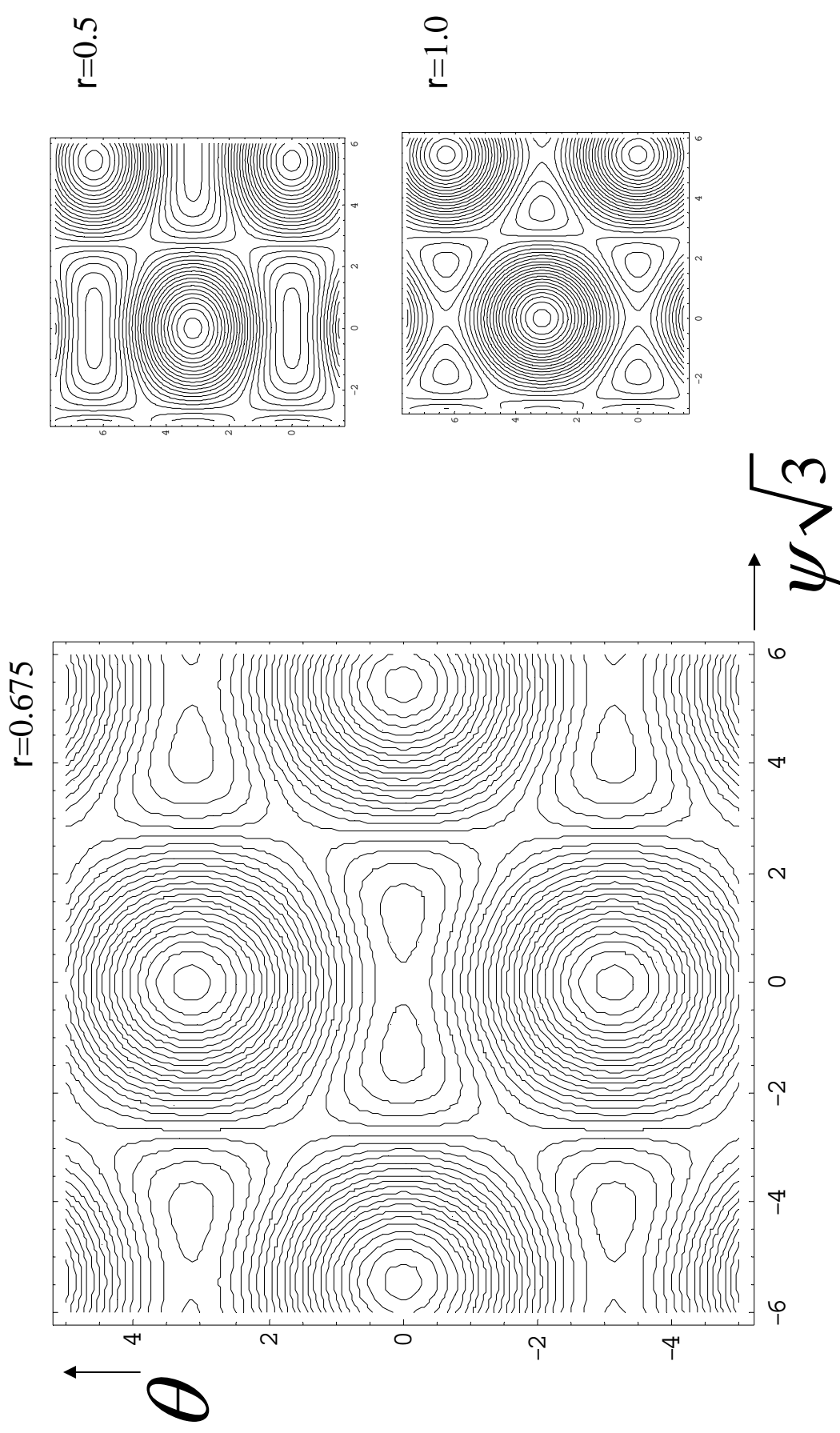


$$\underline{V(\theta, \psi) = -2\{\cos(\theta)\cos(\psi) + \cos(\phi_s)\cos(2\psi - \phi_a)\}}$$

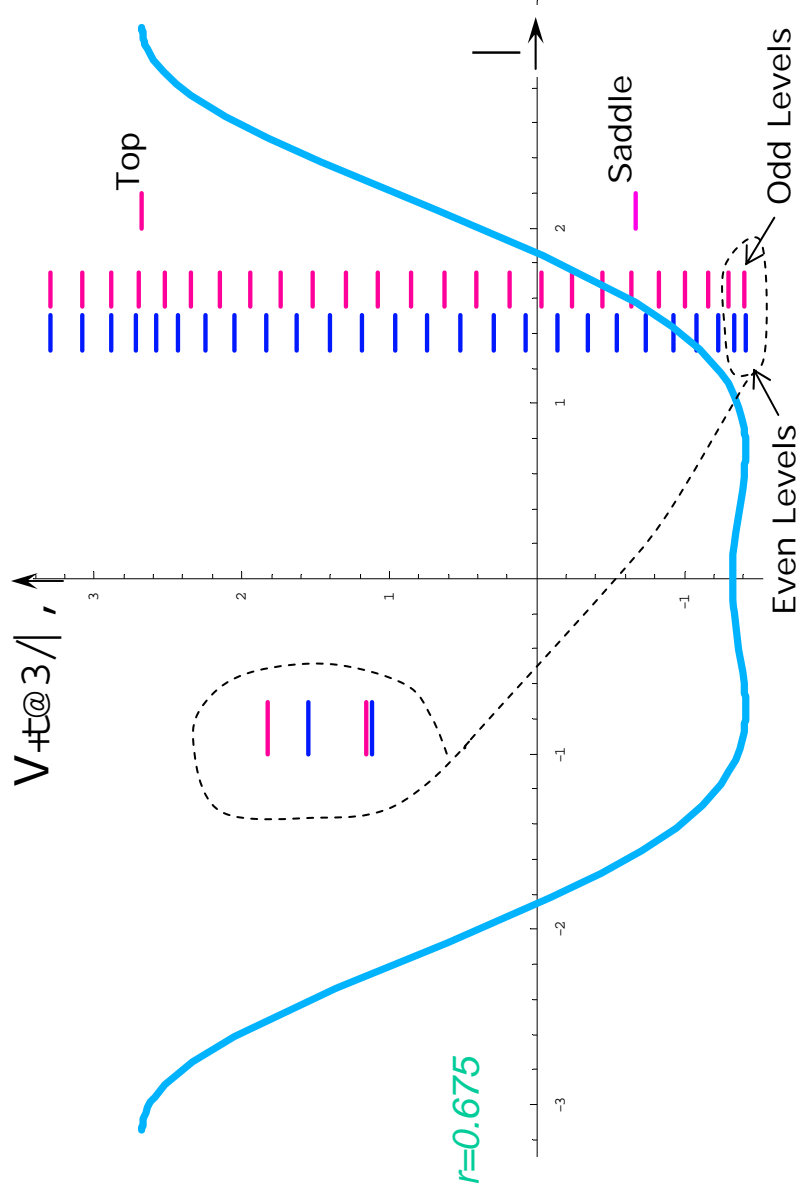
$$\phi_a = (\phi_L - \phi_R) / 2, \quad \theta = (\zeta_1 + \zeta_2) / 2,$$

$$\phi_s = (\phi_L + \phi_R) / 2, \quad \psi = (\zeta_1 - \zeta_2) / 2.$$

Contour Plot of the 2D Potential for the 2 Cell Qubit



Energy Levels for $V(0, |)$



$$-\frac{1}{2m_\psi} \frac{\partial^2 \Psi(\psi)}{\partial \psi^2} - [2 \cos(\psi) - r \cos(2\psi)] \Psi(\psi) = \frac{E_i}{E_J} \Psi(\psi)$$

Circulating Current Patterns

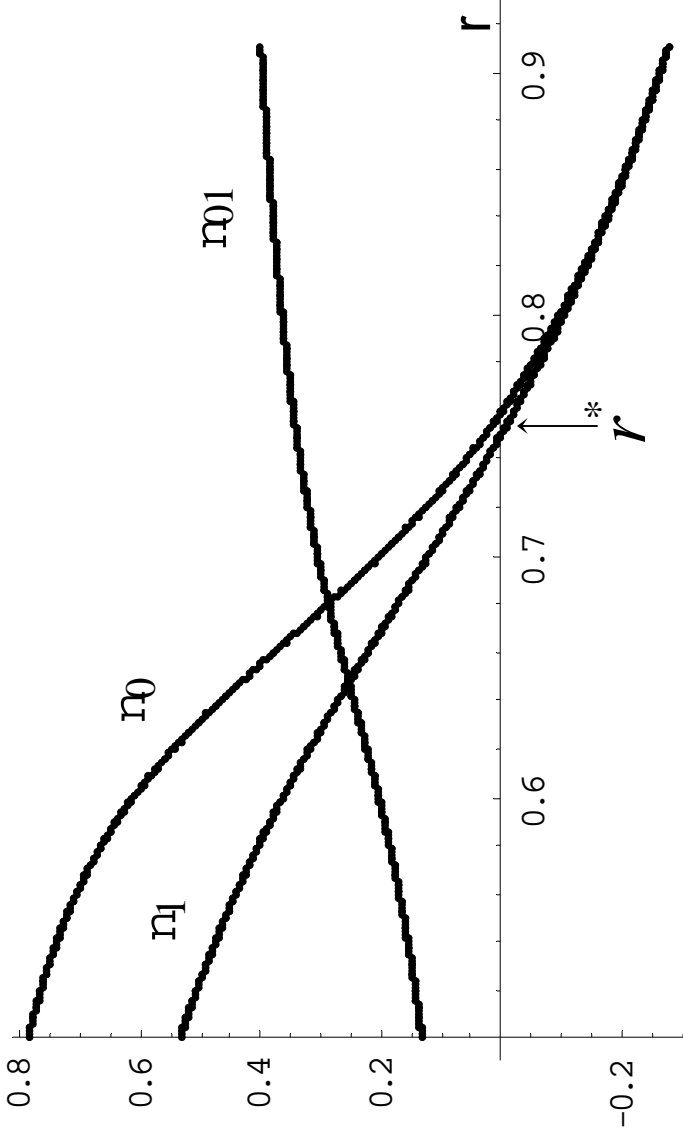
$$J_{3,4}^{|0\rangle} = \langle 0 | J_{C3,4} \sin \zeta_{3,4} | 0 \rangle = \mp j\kappa_0, \quad J_{3,4}^{|1\rangle} = \langle 1 | J_{C3,4} \sin \zeta_{3,4} | 1 \rangle = \mp j\kappa_1$$

$$J_3^{+} = -\frac{1}{2}j(\kappa_0 + \kappa_1) + j\kappa_{01},$$

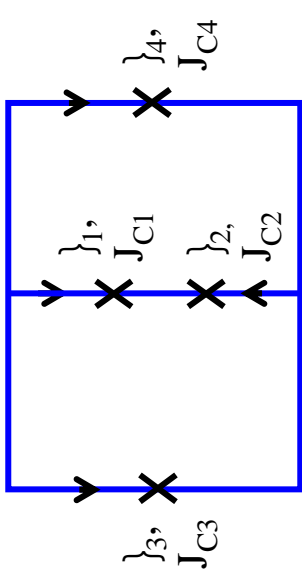
$$J_4^{+} = +\frac{1}{2}j(\kappa_0 + \kappa_1) + j\kappa_{01},$$

$$J_3^{-} = -\frac{1}{2}j(\kappa_0 + \kappa_1) - j\kappa_{01}$$

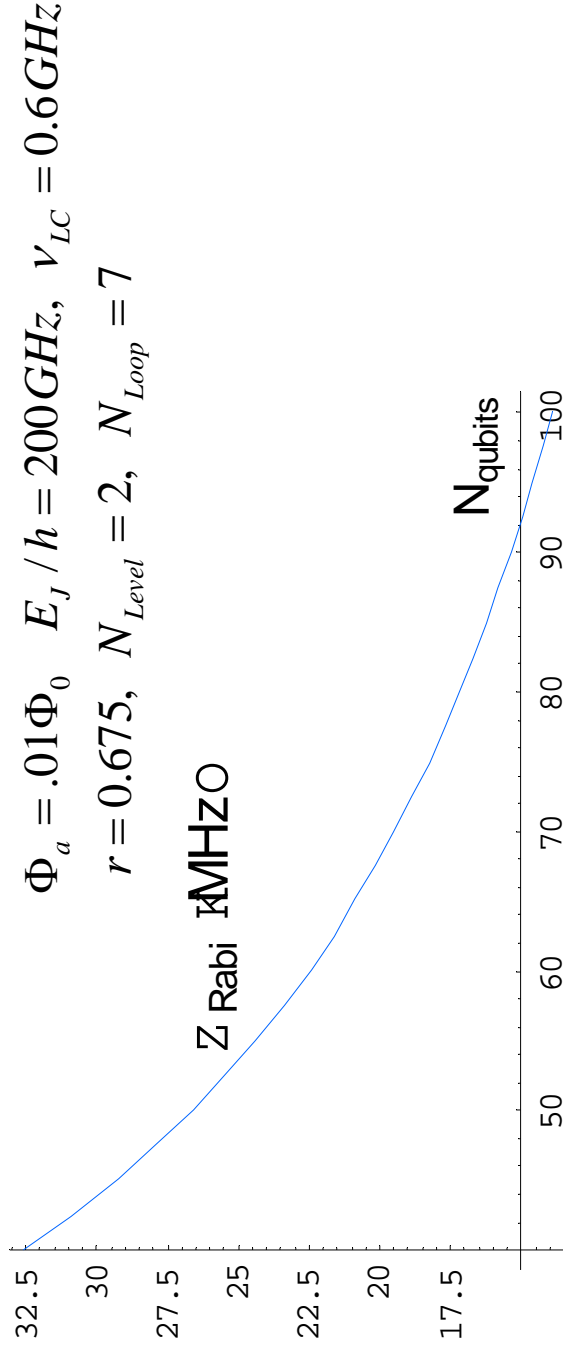
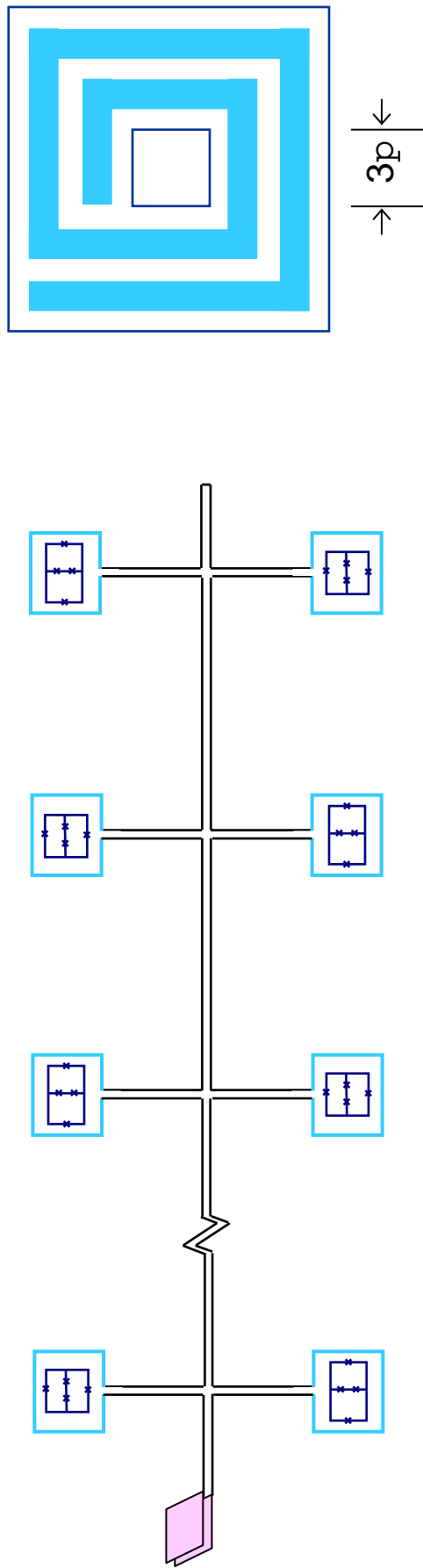
$$J_4^{-} = +\frac{1}{2}j(\kappa_0 + \kappa_1) - j\kappa_{01}.$$



$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

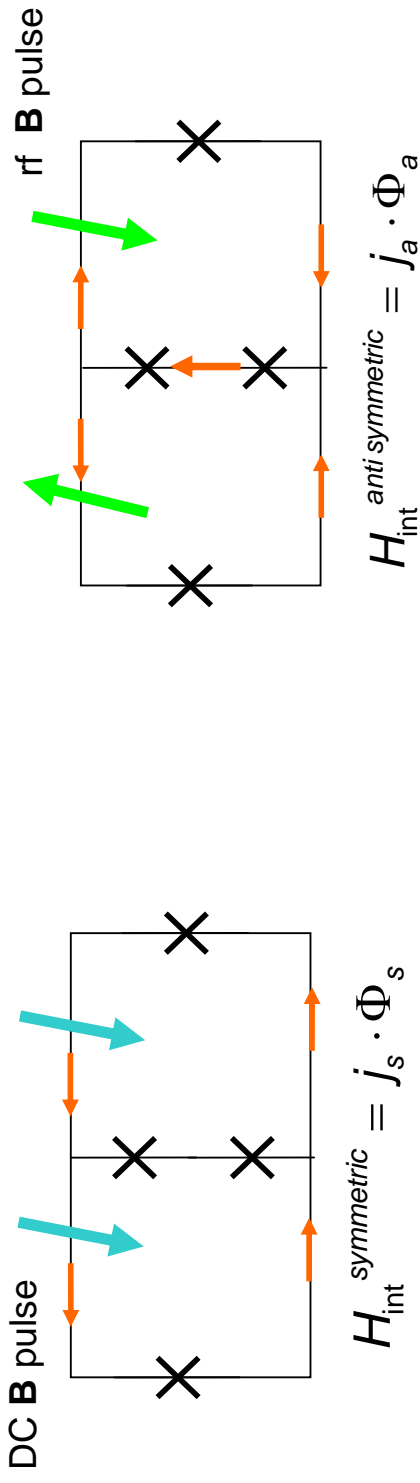


Resonant LC Bus Symmetrically Coupled to Qubit Pairs

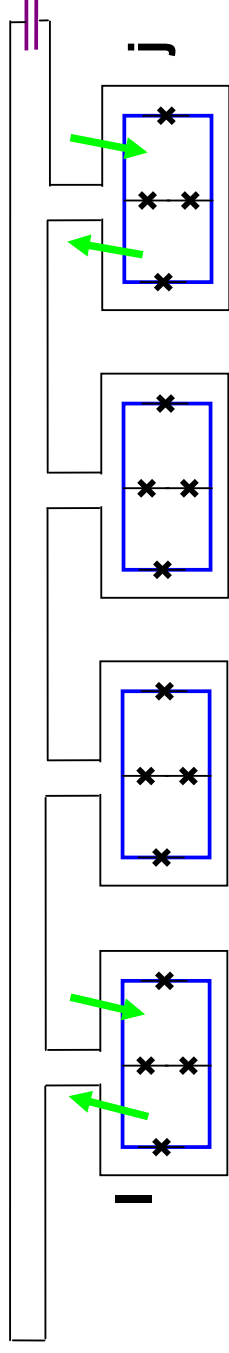


Two Cell Qubit Interactions

First order interactions: yield z and x axis qubit rotations

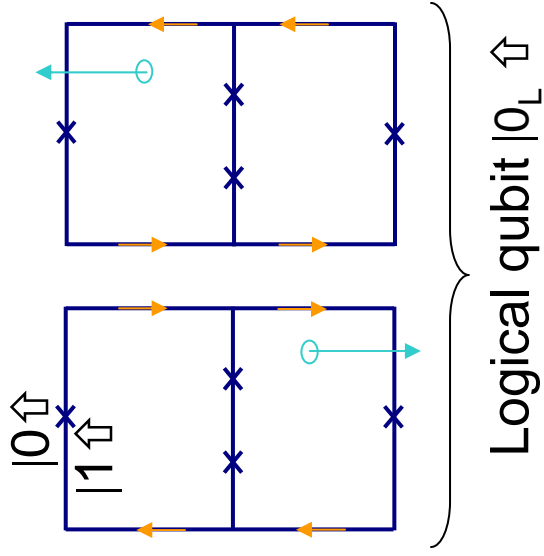


Second order interactions mediated by an off resonance LC bus

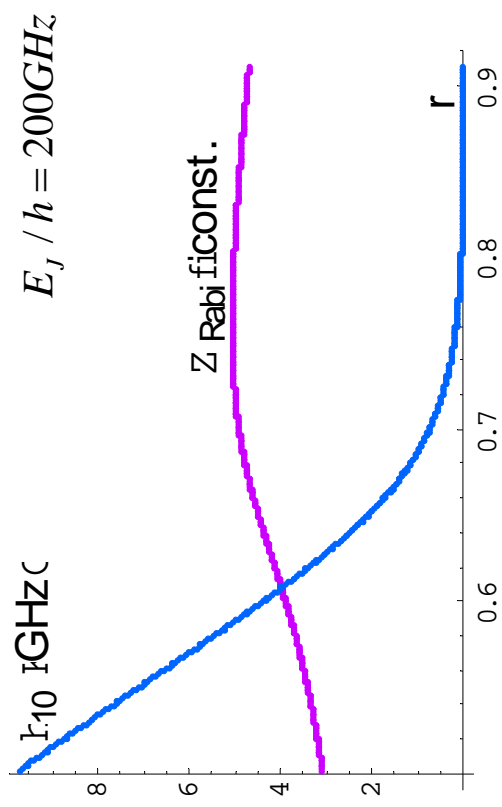
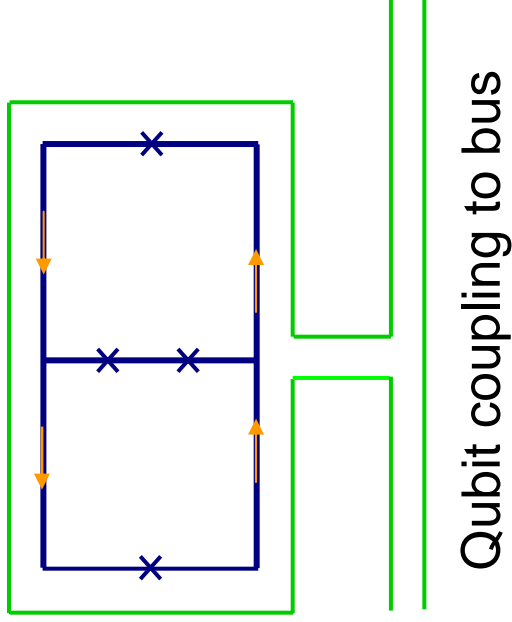
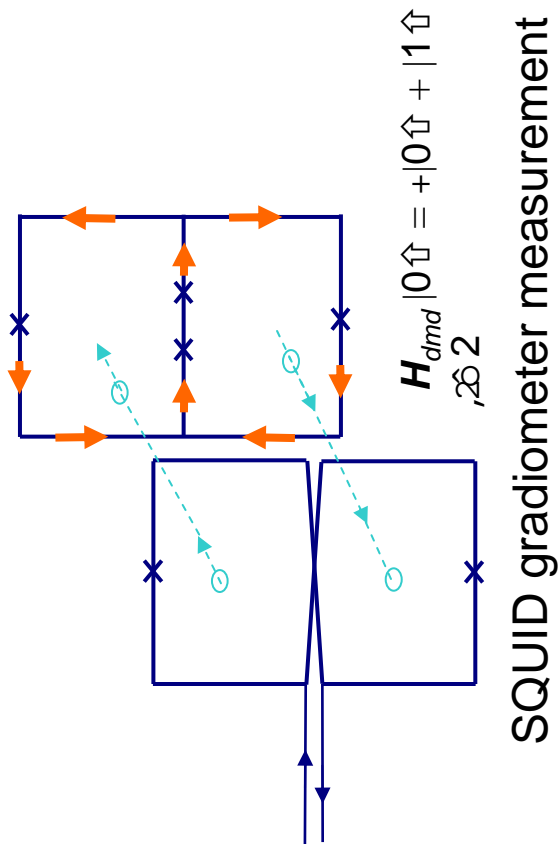


yield $H_{\text{effective}}^{(i,j)} = \Omega_P (\sigma_+^{(i)} \sigma_-^{(j)} + \sigma_-^{(i)} \sigma_+^{(j)})$, an XY spin Hamiltonian interaction

Initializing and Measuring Qubit States



9



Logical Qubits for Decoherence Free Subspaces

The bus-qubit interaction Hamiltonian can be written as

$$H_{\text{int}}^{(i)} = \Omega^{(i)} f(t) \sigma_x^{(i)} (a + a^\dagger)$$

where $f(t)$ is the pulse envelope, and

$$\Omega^{(i)} = [-r \tan(\phi_s^0) \delta_a^{(i)} s_{10} \sqrt{\hbar \omega_{LC} L / 2} \mu(L^{(i)} / 2L)].$$

An unwanted concomitant that will induce gate phase errors is

$$\frac{1}{2} r (c_{00} - c_{11}) \{J_0 [f(t) \delta_a^{(i)}] - 1\} \sigma_z^{(i)} \equiv \delta \Omega^{(i)} \sigma_z^{(i)}. \quad \text{Gate phase errors}$$

and fluctuations of symmetric flux $\sin(\delta \phi_s^{\text{ext}}) \sigma_z^{(i)}$ can be nulled by using either encoded logical product state qubits or Bell state qubits

$$|0_L\rangle = |01\rangle \equiv |0\rangle \otimes |1\rangle, \quad |1_L\rangle = |10\rangle \equiv |1\rangle \otimes |0\rangle$$

$$|\tilde{0}_L\rangle = (|01\rangle + |10\rangle) / \sqrt{2}, \quad |\tilde{1}_L\rangle = (|01\rangle - |10\rangle) / \sqrt{2}.$$

For either type

$$[\delta \Omega^{(i)} \hat{\sigma}_z^{(i)} + \delta \Omega^{(i+1)} \hat{\sigma}_z^{(i+1)}] \left\{ \begin{array}{l} |10\rangle \\ |01\rangle \end{array} \right\} = 0 \quad \text{if } \delta \Omega^{(i)} = \delta \Omega^{(i+1)}.$$

Matrix Elements of Product and Bell Logical Qubits

Bell State qubits have the property that their eigen energies are **exactly** equal

$$\langle \tilde{0}_L | H^{(1)} + H^{(2)} | \tilde{0}_L \rangle = \langle \tilde{1}_L | H^{(1)} + H^{(2)} | \tilde{1}_L \rangle =$$

$$[\langle 0 | H^{(1)} | 0 \rangle + \langle 1 | H^{(1)} | 1 \rangle + \langle 0 | H^{(2)} | 0 \rangle + \langle 1 | H^{(2)} | 1 \rangle]$$

whereas $t_r = \langle \tilde{0}_L | H^{(1)} + H^{(2)} | \tilde{1}_L \rangle = (\varepsilon_1 - \varepsilon_0)^{(1)} - (\varepsilon_1 - \varepsilon_0)^{(2)}$, if not equal to zero will lead to tunneling between the two states.

Transforming to a basis where $t_r = 0$ yields product states where

$$\langle 0_L | H^{(1)} + H^{(2)} | 0_L \rangle - \langle 1_L | H^{(1)} + H^{(2)} | 1_L \rangle = (\varepsilon_1 - \varepsilon_0)^{(2)} - (\varepsilon_1 - \varepsilon_0)^{(1)}.$$

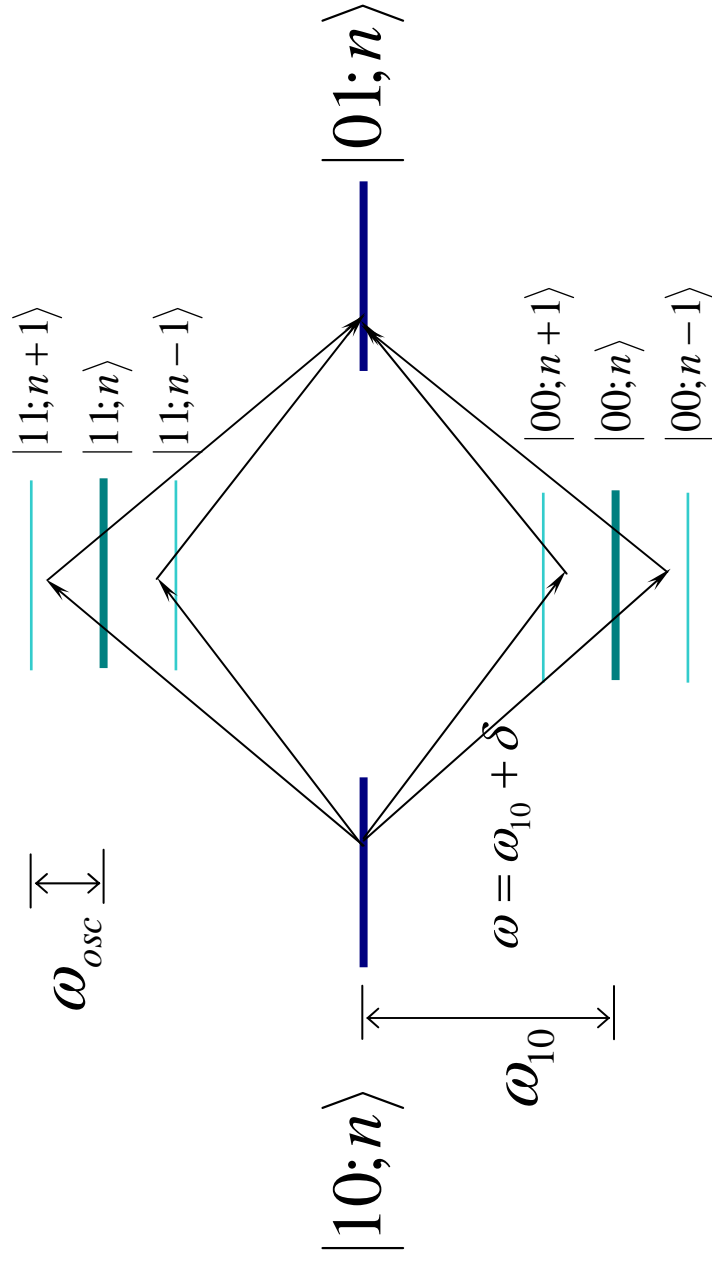
For a QC to have zero tunneling among entangled states it should be much simpler to require the pairwise equality of $(\varepsilon_1 - \varepsilon_0)^{(i)}$ and $(\varepsilon_1 - \varepsilon_0)^{(j)}$ for each logical qubit ($j = i+1$) than global equality for all physical qubits.

Mølmer Sørensen(MS) Gate

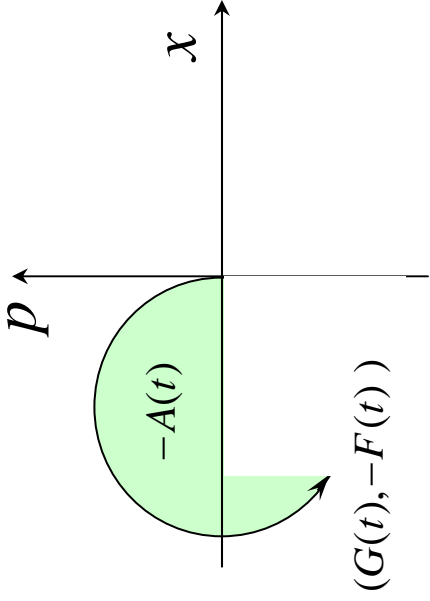
$$\Omega_{eff} = (2/\hbar) \sum_m \frac{\langle 10;n|H_{int}|m\rangle\langle m|H_{int}|01\rangle}{E_m - (E_{01} + \hbar\omega_m)} = -\frac{\Omega^2}{\omega_{osc} - \delta}$$

$$\omega = \omega_{10} \pm \delta$$

Bichromatic
excitation



Mølmer Sørensen Gate II



$$V(t) \cong -\sqrt{2}\Omega J_x(\vec{\phi})[x(\cos(\omega_{LC}-\delta)t + p(\sin(\omega_{LC}-\delta)t) \\ = J_x(\vec{\phi})[xf(t) + pg(t)]$$

The exact solution for $i\dot{U}(t)=VU(t)$ is

$$U(t)=e^{-iA(t)}J_x^2e^{-iF(t)}J_xxe^{-iG(t)}J_xp$$

$$F(t)=\int_0^tf(\tau)d\tau=-\frac{\sqrt{2}\Omega}{(\omega_{LC}-\delta)}\sin[(\omega_{LC}-\delta)t],$$

$$G(t)=\int_0^tg(\tau)d\tau=-\frac{\sqrt{2}\Omega}{(\omega_{LC}-\delta)}\{1-\cos[(\omega_{LC}-\delta)t]\}, \quad A(t)=-\int_0^tF(\tau)g(\tau)d\tau=-\frac{\Omega^2}{(\omega_{LC}-\delta)}\left\{t-\frac{\sin(\omega_{LC}-\delta)t}{2(\omega_{LC}-\delta)}\right\}$$

$U(t)$ performs translations in xp phase space entangled with J_x

$$(x,p)\rightarrow (x+J_xG(t),p-J_xF(t)) \quad J_x=\frac{1}{2}\sum_i[e^{i\phi_i}\sigma_+^i+e^{-i\phi_i}\sigma_-^i]$$

Single Qubit MS Gate Constraints

$$U(t) = e^{-iA(t)J_x^2} e^{-iF(t)J_x} e^{-iG(t)J_x} P$$

For the oscillator variables to return to their pre-gate values

$$\left. \begin{aligned} F(t) &= -\frac{\sqrt{2}\Omega}{(\omega_{LC} - \delta)} \sin[(\omega_{LC} - \delta)t] = 0 \\ G(t) &= -\frac{\sqrt{2}\Omega}{(\omega_{LC} - \delta)} \{1 - \cos[(\omega_{LC} - \delta)t]\} = 0 \end{aligned} \right\} \Rightarrow t_K = K 2\pi / (\omega_{LC} - \delta)$$

$A(t_K)$ is then given by

$$A(t_K) = -\frac{\Omega^2}{(\omega_{LC} - \delta)} t_K$$

These may be combined to give the required pulse time as a function of $A(t_K)$, ω_{LC} , δ , and Ω as

$$t_K = \sqrt{-2\pi K A / \Omega} \quad \text{also} \quad \delta = \omega_{LC} \pm 2\Omega (A_{\pi/2} / A)^{1/2}$$

2 & 4 Qubit Mølmer Sørensen Operators

$$J_x(\vec{\phi}) = \frac{1}{2} \sum_i [e^{i\phi_i} \hat{\sigma}_+^i + e^{-i\phi_i} \hat{\sigma}_-^i] \lambda_i, \quad \lambda_i = (\Omega_i / \Omega)$$

$$2J_x^2 = \frac{1}{2}(\lambda_1^2 + \lambda_2^2) \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} + \lambda_1 \lambda_2 \begin{bmatrix} & & & e^{+i(\phi_1 + \phi_2)} \\ & & e^{+i(\phi_1 - \phi_2)} & \\ & e^{-i(\phi_1 - \phi_2)} & & \\ e^{-i(\phi_1 + \phi_2)} & & & \end{bmatrix}$$

Single logical

$$\bar{X} = 2J_x^2(\phi_{1,2} = 0) = \frac{1}{2}(\lambda_1^2 + \lambda_2^2) \hat{1} + \lambda_1 \lambda_2 \hat{\sigma}_x^{(1)} \hat{\sigma}_x^{(2)},$$

qubit operators

$$\bar{Y} = 2J_x^2(\phi_1 = 0, \phi_2 = \frac{1}{2}\pi) = \frac{1}{2}(\lambda_1^2 + \lambda_2^2) \hat{1} - \lambda_1 \lambda_2 \hat{\sigma}_x^{(1)} \hat{\sigma}_y^{(2)}$$

Two logical

$$\overline{XX} = 2J_x^2(\phi_{1,2,3,4} = 0) = \frac{1}{2} \sum_{i=1}^4 \lambda_i^2 \hat{1} + \sum_{i=1}^4 \sum_{j=i+1}^4 \lambda_i \lambda_j \hat{\sigma}_x^{(i)} \hat{\sigma}_x^{(j)}$$

qubit operators

$$\lambda_j = 1 \rightarrow \exp(i \frac{\pi}{4} \overline{XX}) = e^{-i \frac{\pi}{4}} \exp[-i \frac{\pi}{4} \hat{\sigma}_x^{(1)} \hat{\sigma}_x^{(2)} \hat{\sigma}_x^{(3)} \hat{\sigma}_x^{(4)}].$$

One and Two Logical Qubit Mølmer Sørensen Gates

Product logical qubits

Bell state logical qubits

$$|0_L\rangle = |01\rangle, \quad |1_L\rangle = |10\rangle \quad \left| \tilde{0}_L, \tilde{1}_L \right\rangle = \frac{1}{\sqrt{2}} \{ |0_L\rangle \pm |1_L\rangle \}$$

$\sigma_{Lx} = \sigma_x^{(1)} \sigma_x^{(2)}$	$\tilde{\sigma}_{Lz} = \sigma_x^{(1)} \sigma_x^{(2)}$
$\sigma_{Ly} = \sigma_y^{(1)} \sigma_x^{(2)}$	$\tilde{\sigma}_{Ly} = -\sigma_y^{(1)} \sigma_x^{(2)}$
$\sigma_{Lz} = -i \sigma_{Lx} \sigma_{Ly}$	$\tilde{\sigma}_{Lx} = -i \tilde{\sigma}_{Lx} \tilde{\sigma}_{Ly}$
$U_{CN}^{I,\Pi} = \tilde{U}_{CN}^{\Pi,I}$ $U_{CZ}^{I,\Pi} = H^{\Pi} U_{CN}^{I,\Pi} H^{\Pi}$	$\tilde{U}_{CZ}^{I,\Pi} = \sqrt{i} e^{i\tilde{\sigma}_z^{(I)} \tilde{\sigma}_z^{(\Pi)} \pi/4} e^{-i\tilde{\sigma}_z^{(I)} \pi/4} e^{-i\tilde{\sigma}_z^{(\Pi)} \pi/4}$ $\tilde{U}_{CN}^{I,\Pi} = \tilde{H}^{\Pi} \tilde{U}_{CZ}^{I,\Pi} \tilde{H}^{\Pi}$

where $H_L = \tilde{H}_L = (-i) e^{-i\tilde{\sigma}_{Ly} \pi/4} e^{+i\tilde{\sigma}_{Lz} \pi/2}$

Critical Current and Cell Geometry Defects

$$\Delta V = -\{2j \sin \phi_{a,geom} \cos \phi_s^0 + 2\Delta j \cos \phi_{a,geom} \sin \phi_s^0\} \sin 2\psi - 2\Delta J \sin \theta \sin \psi$$

$$= -\kappa_{defect} \sin 2\psi - 2\Delta J \sin \theta \sin \psi = \Delta V^a + \Delta V^b$$

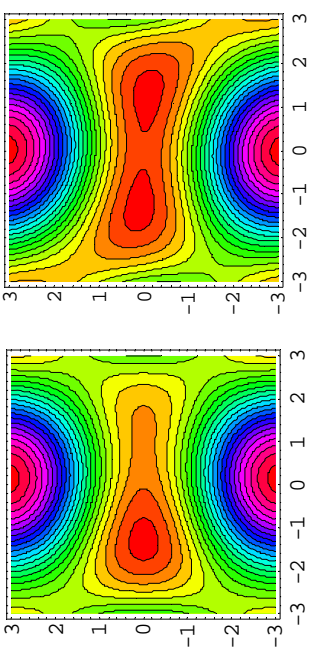
ΔV^a may be nulled by adding a small trim flux so that $\phi_a^0 = \phi_{a,geom} + \phi_{a,trim}$ satisfies

$$\tan \phi_a^0 = -(\Delta j / j) \tan \phi_s^0 \Rightarrow \{\Delta V^a = 0,$$

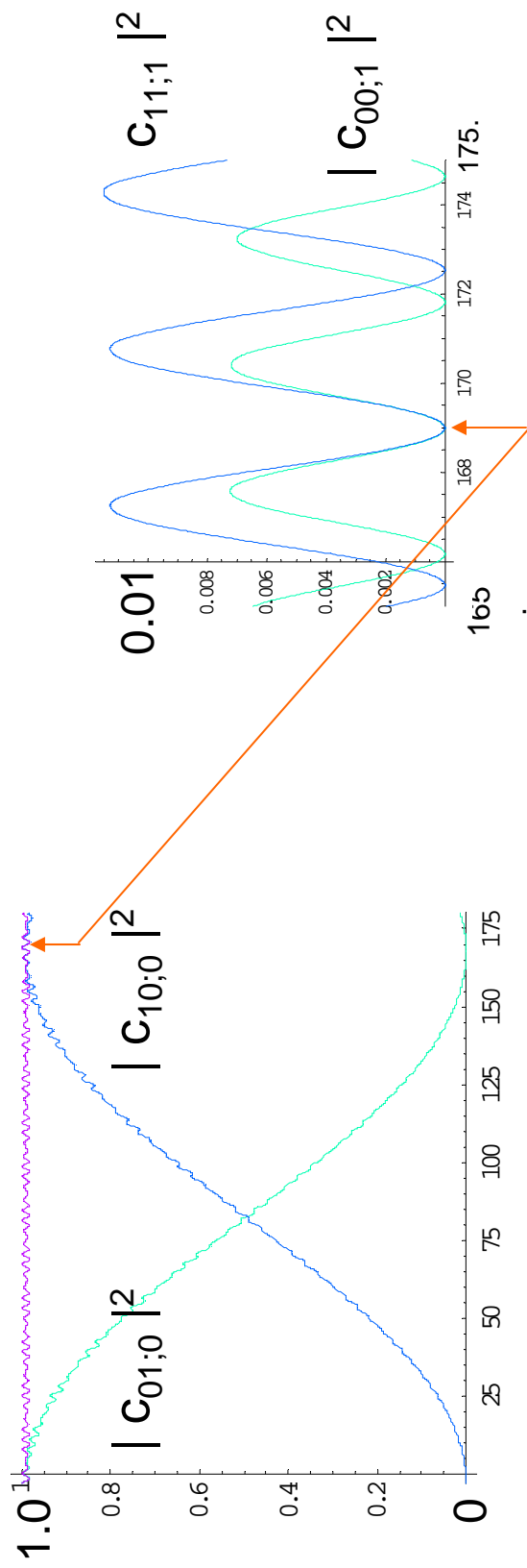
$J_{3,4}^{[0,1]} \rightarrow J_{3,4}^{[0,1]} \times (1 - (\Delta j / j)^2)$, figure 8 currents stay zero, $r \rightarrow r / \cos \phi_a^0 \}$

If $\phi_{a,trim}$ is not added, $H_0 + \Delta V_a$ can be diagonalized by a unitary rotation to a new basis but there will be a non zero figure 8 current $\kappa_{0,1} j (\Delta j / j + \tan \phi_{a,geom} / \tan \phi_s^0)$. At $r=r^*$, $\kappa_{0,1} (\Delta j / j) \approx 10^{-4}$.

The effect of ΔV^b can be estimated by perturbation theory. The unperturbed states $|0\rangle$ and $|1\rangle$ will pick up small admixtures of $|1_\theta\rangle|1_\psi\rangle$ and $|1_\theta\rangle|0_\psi\rangle$ respectively, but this will not lead to figure 8 currents;

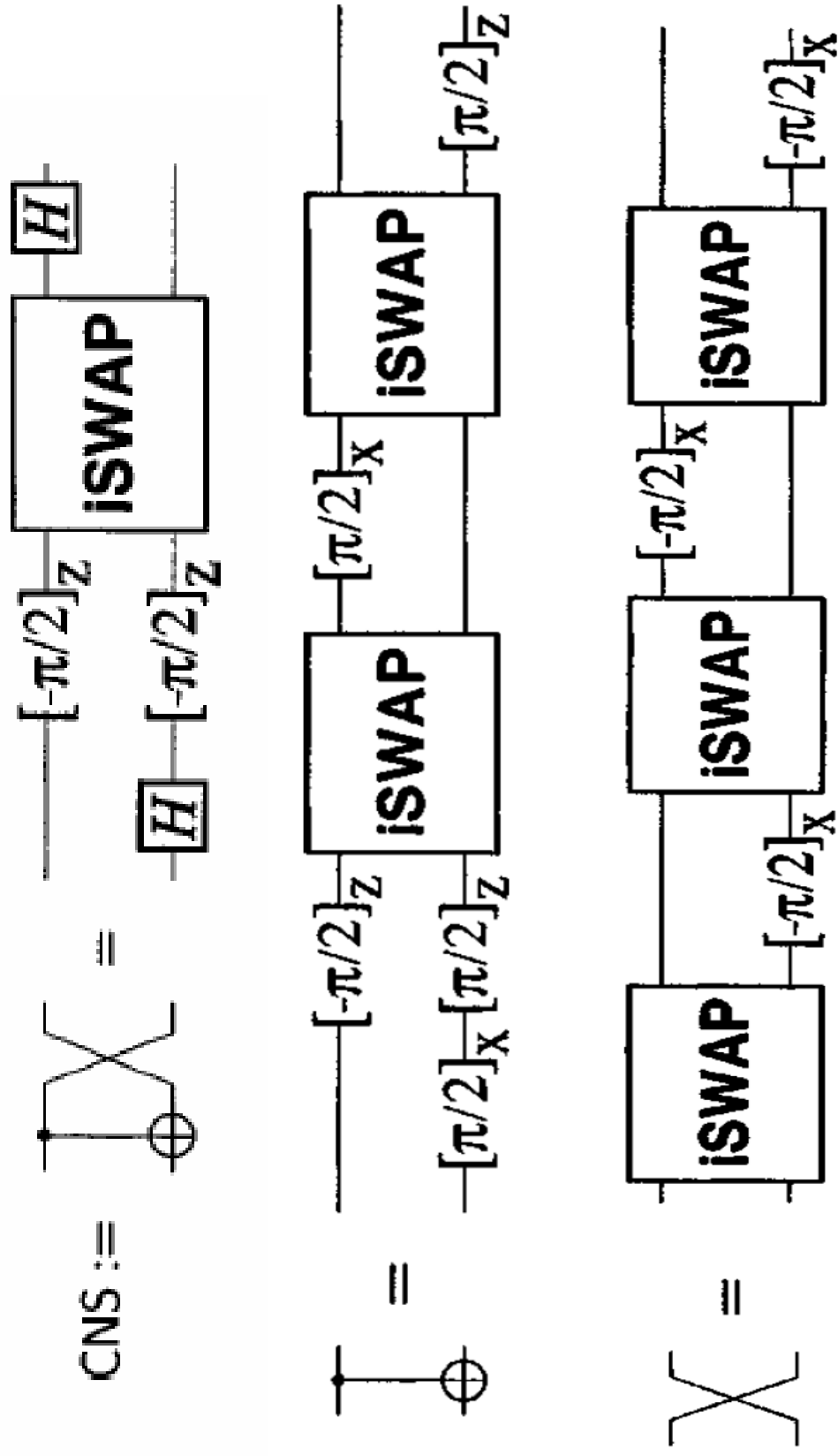


Numerical Solution for iSWAP Gate

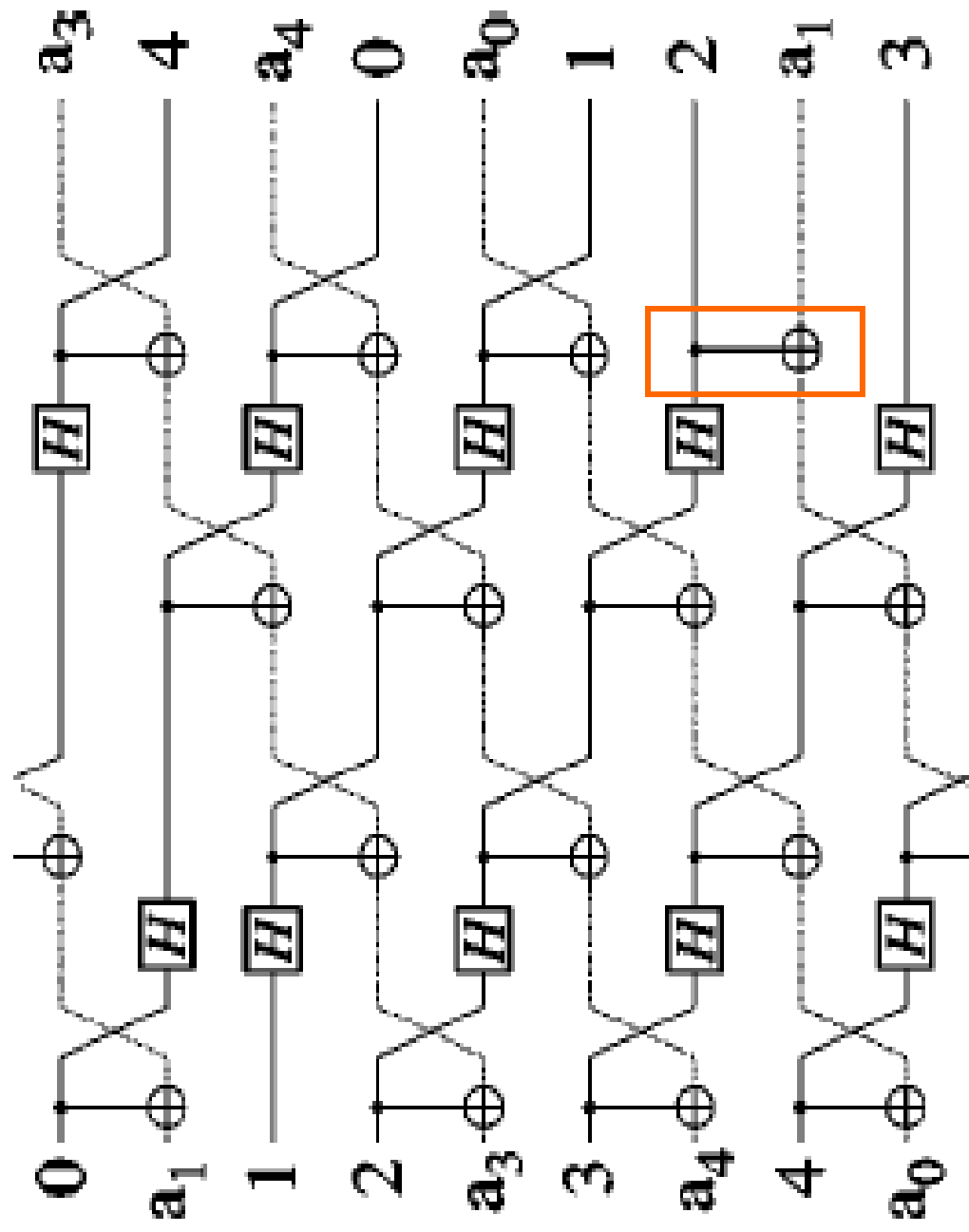


$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \equiv iSWAP, \quad |c\rangle = \begin{bmatrix} c_{00}|00\rangle \\ c_{01}|01\rangle \\ c_{10}|10\rangle \\ c_{11}|11\rangle \end{bmatrix}$$

CNOT and SWAP Gates from iSwap

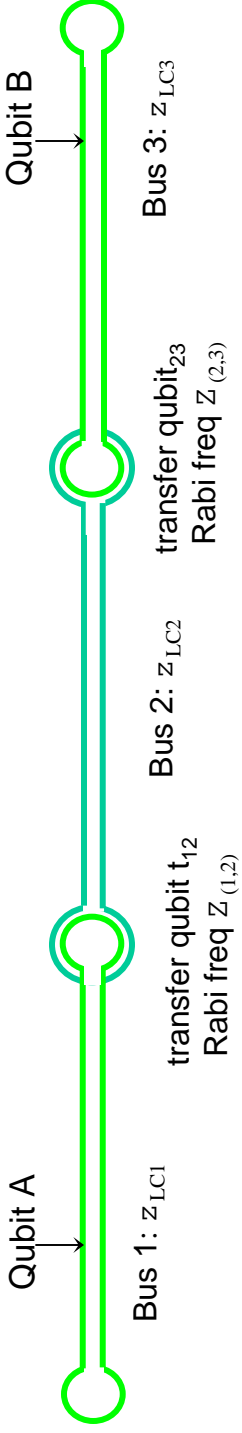


Five Qubit Error Correcting Code



Forming a CN Gate Across a Network

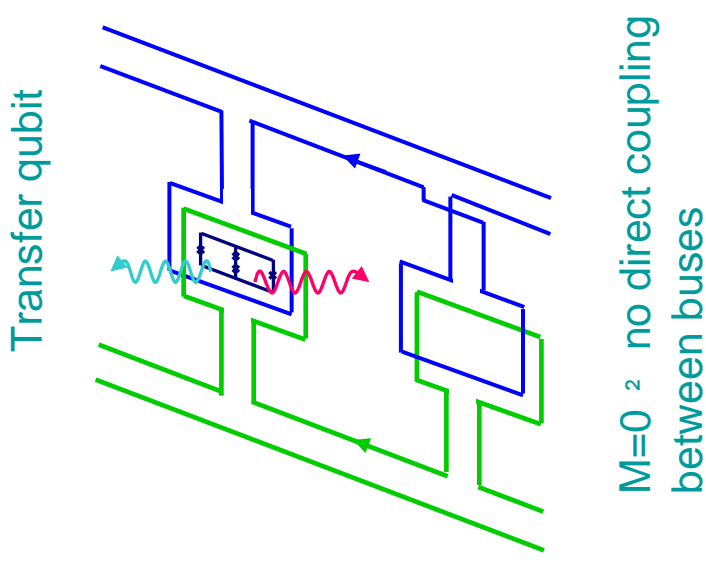
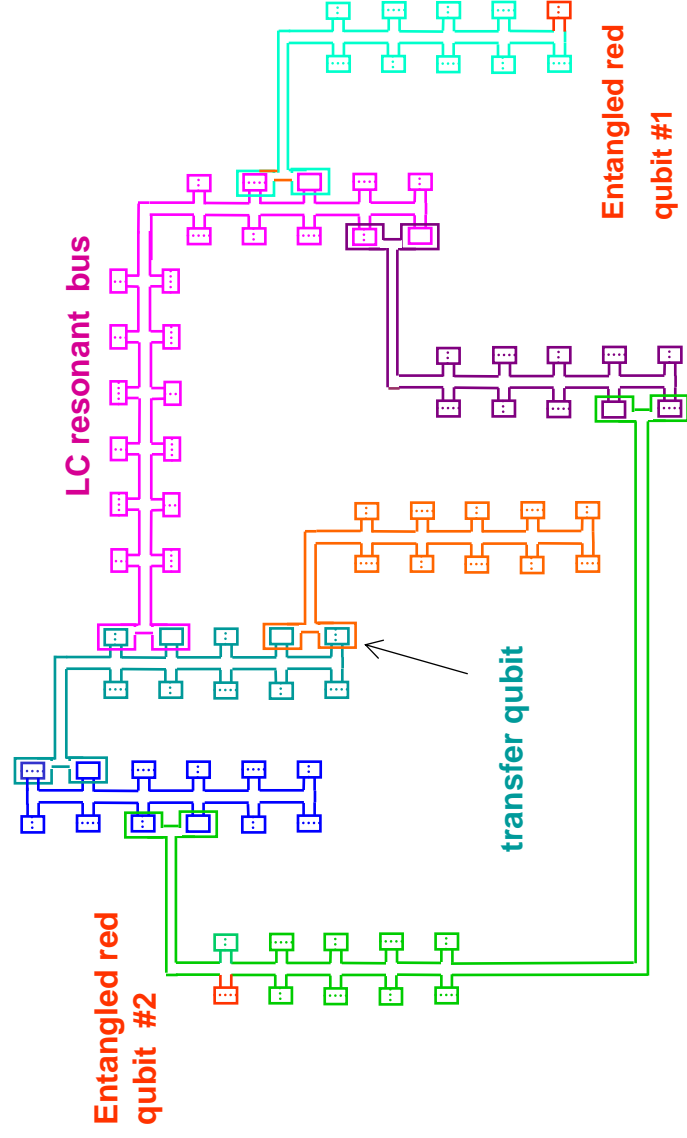
$$\tilde{U}_{CN}^{I,II} = \sqrt{i} e^{-i\tilde{\sigma}_y^{(II)}\pi/4} e^{-i\tilde{\sigma}_z^{(I)}\pi/4} e^{i\tilde{\sigma}_z^{(I)}\tilde{\sigma}_z^{(II)}\pi/4} e^{-i\tilde{\sigma}_z^{(II)}\pi/4} e^{+i\tilde{\sigma}_y^{(II)}\pi/4}$$



For $\tilde{U}_{CN}^{A,B}$ need a sequence of CN gates $(A, t_{12}), (t_{12}, t_{23}), (t_{23}, B)$ and a sequence $(A, t_{12}), (t_{12}, t_{23})$ to reset the transfer qubits.

For every transfer qubit involved, one less single qubit gate $\tilde{\sigma}_z$ is needed, reducing the total gate time by $T_{1/2} = \nu_{Rabi}^{-1} / 2$ for each instance. Thus the time needed for (t_{12}, t_{23}) is $3T_{1/2}$.

Coupling Buses into a Network



Coupling buses into a network using transfer qubits enables:

- Simultaneous (intra-bus) two qubit gates
- Scaling to arbitrary qubit numbers
- Teletransport of qubits via entangled qubit pairs

Two Readout Squid Circuit

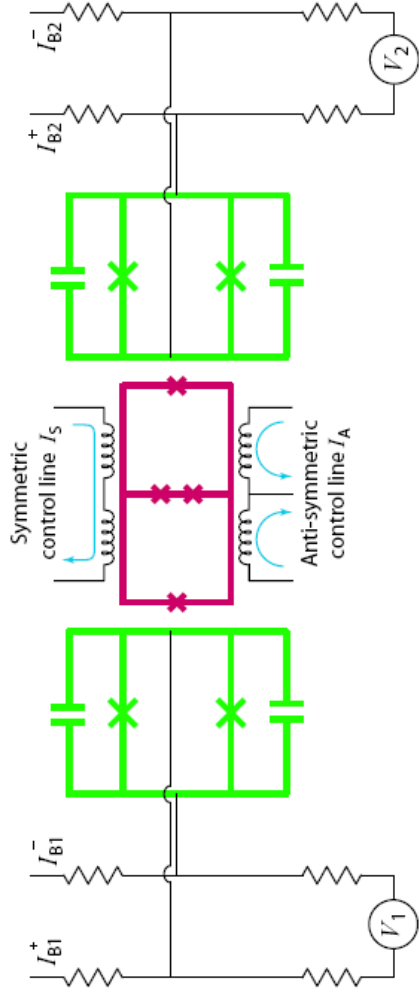


Figure 9: Schematic circuit of a qubit inductively coupled to a readout dc-SQUID. Each SQUID is shunted by two 5 pF capacitor. The current bias and voltage measurements are performed via 150 Ohm resistor on each lead.

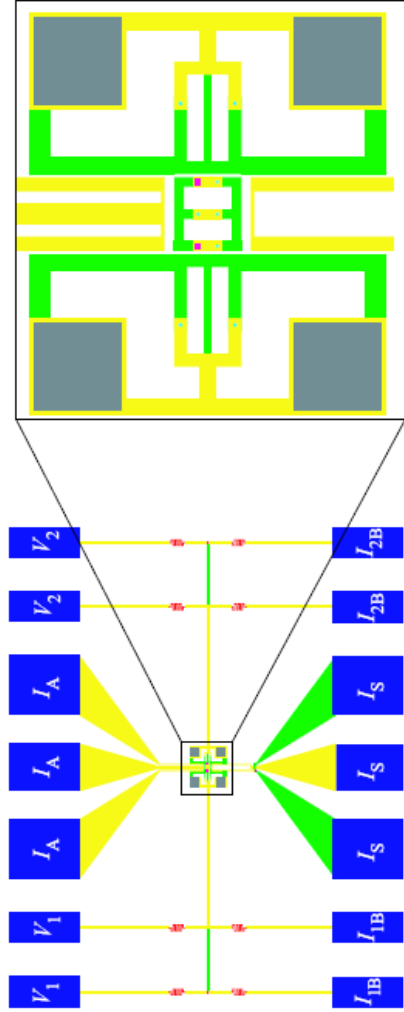


Figure 8: Micro design of inductively coupled qubit and two SQUID.

Gradiometer Coupling to SQUID

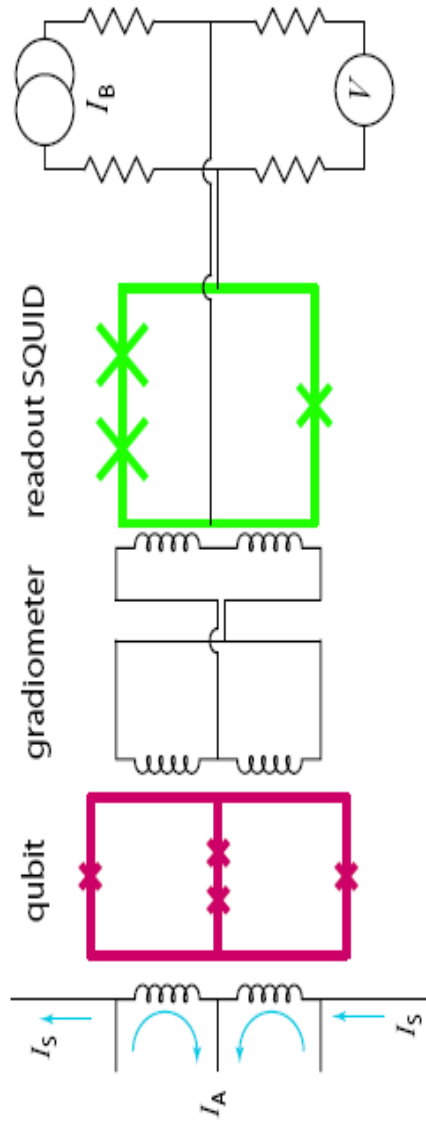


Figure 12: Schematic circuit of a qubit coupled to a readout dc-SQUID via a gradiometric loop. The SQUID is asymmetric.

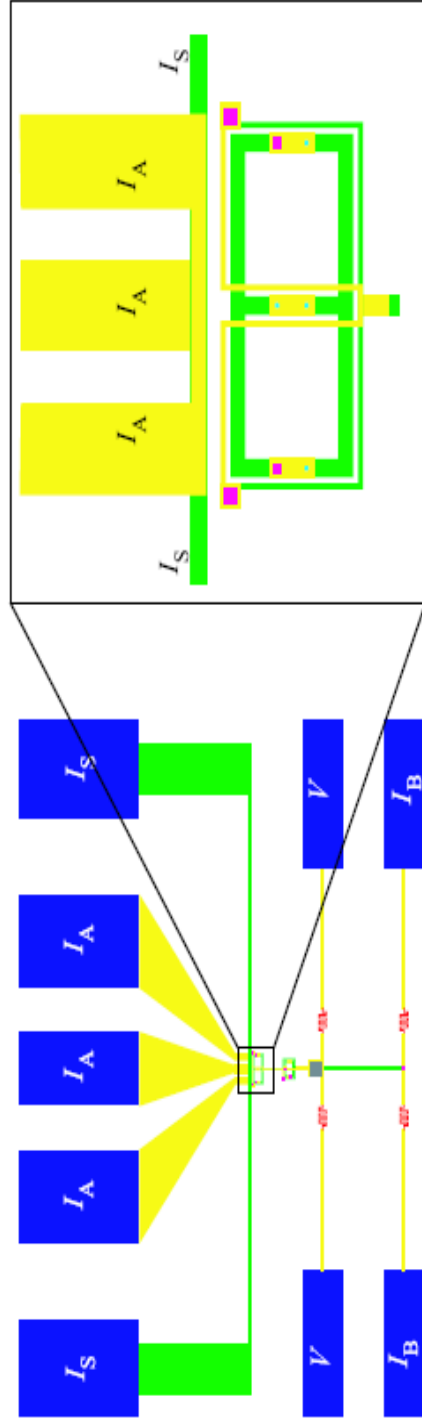


Figure 13: The actual design of gradiometric coupling between a qubit and a SQUID.

Conclusions

Using logical prism qubit pairs and MS gates it is possible to carry out all necessary gates for quantum computation:

- 1) Pairwise adjustment of qubit transition frequencies (by $\phi_s^{0(i)}$) to obtain $\mathcal{E}_{10}^{(i)} = \mathcal{E}_{10}^{(i+1)}$ can enable global phase coherence (no time evolution or decay of entangled states).
- 2) Pairwise adjustment of Rabi frequencies $\Omega_{Rabi}^{(i)} = \Omega_{Rabi}^{(i+1)}$ (by adjusting $\delta_a^{(i)}$) enables MS gates to be carried out without phase errors. All two qubit gates can be carried out when timing is adjusted to compensate for $\Omega_{Rabi}^{(A)} \neq \Omega_{Rabi}^{(B)}$.
- 3) Only qubit perimeter currents exist during a computation; there are no figure 8 currents to interact with SQUID gradiometer detectors; DFS properties are preserved even during MS gates.
- 4) If the resonant frequencies of adjacent buses differ by $\square 4\Omega_{Rabi}$ Molmer Sorensen gates can be carried out across an open network by employing transfer qubits and chaining CN gates.